Landau theory of the Fermi-liquid to electron glass transition

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A lattice model of spinless interacting electrons is used to formulate the Landau theory of the Fermi liquid to electron glass quantum phase transition. We demonstrate that the presence of additional random site energies does not affect the character of the transition, once the replica symmetry breaking is considered self-consistently at the mean-field level. Inside the glass phase, the low temperature conductivity assumes a non-Fermi liquid $\delta\sigma \sim T^{3/2}$ form, in agreement with recent experiments.

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Numerous recent experiments have demonstrated that the phase diagram of the low carrier density systems proves remarkably rich even in the absence of magnetic fields [1]. The fascinating strong correlation physics has been observed whenever the energy scale of the Coulomb repulsive interactions is comparable to that of the Fermi energy. Once disorder is weak enough, the interactions alone can lead to insulating behavior, characterized by hard gaps in the charge sector. But if the magnitude of random potential is comparable to the Fermi energy, electrons approach the strongly localized regime. In this case, the interplay between localization and Coulomb interactions is generally expected to lead to (gapless) glassy ordering of electrons [2, 3].

Very interesting new information on this problem comes from recent experiments by Bogdanovich and Popović [4]. In this work, strong long-time fluctuations of conductivity were observed as the electron density is reduced below some critical value, indicating a dramatic slowing down of electron dynamics. This behavior was attributed to the presence of the glassy freezing that appears to come from the charge degrees of freedom. Interestingly, the glassy behavior seems to emerge appreciably before the metal-insulator transition, thus identifying an intermediate metallic glass phase. Inside the glass phase, unusual temperature dependence of the conductivity was observed, which was fitted to the form $\delta\sigma \sim T^{3/2}$.

The main goal of the current Communication is to theoretically examine the nature of the quantum phase transition from a Fermi liquid to such a metallic glass phase, and the account for the resulting modifications of charge transport. Previous works have extensively examined the quantum paramagnet to spin-glass transitions, regarding thus the spin degrees of freedom as the only relevant ones at criticality[5, 6, 7, 8]. Here, we use a similar approach to study the onset of glassiness in the charge sector. Using a recently developed dynamical mean-field formulation [3], we construct a Landau theory which provides a complete description of the electron dynamics near the relevant quantum critical point.

We consider a lattice model of spinless interacting electrons at half-filling in the presence of on-site randomness, as given by the Hamiltonian

$$H = \sum_{ij}(-t_{ij} + \varepsilon_i\delta_{ij})c_i^\dagger c_j + \sum_{ij} V_{ij} c_i^\dagger c_i c_j^\dagger c_j. \qquad (1)$$

Here t_{ij} denotes the corresponding hopping elements, while V_{ij} describes the inter-site Coulomb interactions. The distribution of the random site energies ε_i is assumed to be Gaussian, with variance W. As the simplest model [3] for the glassy freezing of electrons, we choose the inter-site interactions V_{ij} also to be Gaussian distributed random [9] variables, with variance V . For this model, it is straightforward [3] to employ the methods of dynamical mean-field theory [10] which is formally exact in the limit of large coordination. The averages over randomness are carried out with using standard replica methods [3, 11], which are also used to identify the emergence of the glassy phase. This procedure leads [3] to the following single site effective action ($\omega_m = 2\pi(m+1)T$)

$$S_{\rm eff}(i) = \sum_{\omega_m}\sum_a c_i^{\dagger a}(\omega_m)\left[i\omega_m + t^2 G(\omega_m)\right] c_i^a(\omega_m)$$
$$- \int_0^\beta\int_0^\beta d\tau_1 d\tau_2 \left\{ \frac{V^2}{2}\sum_{ab}\delta n_i^a(\tau_1) Q^{ab}(\tau_1-\tau_2)\delta n_i^b(\tau_2) \right.$$
$$\left. + \frac{W^2}{2}\sum_{ab}\delta n_i^a(\tau_1)\delta n_i^b(\tau_2)\right\} \qquad (2)$$

In the formula above, the functional integration is performed over the fermionic fields $c_i^a(\tau)$ ($a = 1, ..., n$ are the standard replica indices). $\delta n_i^a(\tau) = c_i^{\dagger a}(\tau)c_i^a(\tau) - \frac{1}{2}$ denotes the deviation of the density from half-filling. We took into account that the "Weiss" (cavity) field has the form $W_s(\tau_1-\tau_2) = t^2 G(\tau_1-\tau_2)$ for electrons on a Bethe lattice. The Green function $G(\tau_1-\tau_2)$ along with the order parameter $Q^{ab}(\tau_1-\tau_2)$ must be determined self-consistently using the effective action given by Eq. (2)[3].

$$G(\tau_1-\tau_2) = < c_i^{\dagger a}(\tau_1)c_i^a(\tau_2) >_{\rm eff}, \qquad (3)$$
$$Q^{ab}(\tau_1-\tau_2) = < \delta n_i^a(\tau_1)\delta n_i^b(\tau_2) >_{\rm eff} . \qquad (4)$$

The replica diagonal components of $Q^{aa}(\tau)$ represent the averaged dynamic compressibility, while the parameters

Q^{ab} ($a \neq b$) are time independent and related to the familiar Edwards-Anderson (EA) order parameter[5]. Simple analysis of Eqs. (2)-(4) shows that q_{ab} is non-zero everywhere, once $W \neq 0$. This is the consequence of the non-uniform density due to the on-site random potential. As a result, the glass transition for $W \neq 0$ assumes the character of a De Almeida-Thouless line [3, 11], where special care is needed in formulating a Landau theory. As it will be clear from below, the relevant Landau theory can be rigorously formulated, only if W is small, but our conclusions remain qualitatively correct for arbitrarily large W. If V is much smaller than t, action Eq. (2) describes the usual disordered Fermi liquid[13], while in the opposite limit the glassy ordering persists down to $T = 0$[3]. The quantum phase transition to the glassy phase occurs in this model at some critical value $(t/V)_{\rm cr}$, which has weak dependence on W [3].

To obtain the Landau functional we must perform a cumulant expansion in Eq. (2) treating the term with $Q^{ab}(\tau_1 - \tau_2)$ as perturbation. Before doing this, it is necessary to shift the Q-matrices, eliminating thus the non-critical regular part in diagonal elements ($\beta = 1/T$):

$$Q^{ab}(\omega_n) \to Q^{ab}(\omega_n) - \mathcal{K}\delta^{ab} - \beta\frac{W^2}{V^2}\delta_{\omega_n,0}. \qquad (5)$$

Constant $\mathcal{K}$ must be formally determined from the condition of absence of the term $\sum_{\omega_n}\sum_{ab}[Q^{ab}(\omega_n)]^2$ in the underlying Ginzburg-Landau action[5], that reads:

$$\beta\mathcal{F} = \sum_{a,\omega_n}\left(\frac{r+|\omega_n|}{V^2}\right)Q^{aa}(\omega_n) + \frac{u}{2\beta}\sum_a\left[\sum_{\omega_n}Q^{aa}(\omega_n)\right]^2$$
$$-\frac{V^3}{3}\sum_{abc}\sum_{\omega_n}Q^{ab}(\omega_n)Q^{bc}(\omega_n)Q^{ca}(\omega_n) - \frac{\beta W^2}{2}Q^{ab}(\omega_n = 0)$$
$$-\frac{\beta y}{6}\int\int d\tau_1 d\tau_2 \sum_{ab}\left[Q^{ab}(\tau_1-\tau_2)\right]^4. \qquad (6)$$

Here r, being some function of t/V, is the parameter that governs the transition, while u and y are taken at $(t/V)_{\rm cr}$. The presence of the last term, responsible for the RSB instability, is crucial to further analysis. Accordingly, we employ the following mean-field ansatz for the Q-matrices:

$$V^2Q^{ab}(\omega_n) = \begin{cases} D(\omega_n) + \beta q_{\rm EA}\delta_{\omega_n,0}, & a = b, \\ \beta q_{ab}\delta_{\omega_n,0}, & a \neq b. \end{cases} \qquad (7)$$

In Eq. (7) $q_{\rm EA}$ is the EA order parameter, and it is assumed that $q_{aa} = 0$. The β- prefactors are chosen to ensure the finite limit of the free energy density as $T \to 0$. We must insert Eq. (7) into the action Eq. (6) and find the saddle point solution with respect to the variations of q_{ab}, $q_{\rm EA}$ and $D(\omega_n)$. $q_{\rm EA}$ and q_{ab} should not, however, be varied independently [12]. They must obey an additional relation between them that depends on the presence of the glassy ordering and, hence, the replica symmetry breaking. To make this point clear we first identify the part of the action that contains only q_{ab}:

$$\mathcal{F}_1 = -R_1{\rm Tr}q^2 - \frac{R_2}{3}{\rm Tr}q^3 - \frac{R_3}{6}\sum_{a\neq b}(q_{ab})^4 - R_4\sum_{a\neq b}q_{ab}, \qquad (8)$$

where,

$$R_1 = \beta(D(0) + \beta q_{\rm EA}), \qquad R_2 = \beta^2$$
$$R_3 = \frac{\beta y}{V^4}, \qquad R_4 = \frac{\beta W^2V^2}{2}. \qquad (9)$$

Fermi liquid phase. As we emphasized previously, there is no replica symmetry breaking in the Fermi liquid phase. Therefore, it is natural to choose the parametrization $q_{ab} = q_{\rm EA}$ in this phase and take the variational derivative of Eq. (8) with respect to $q_{\rm EA}$, obtaining thus:

$$2D(0)q_{\rm EA} + \frac{2y}{3V^4}q_{\rm EA}^3 + \frac{W^2V^2}{2} = 0 \qquad (10)$$

Varying subsequently Eq. (6) with respect to $D(\omega_n)$ we arrive at the equation:

$$r + |\omega_n| + u\left[\frac{1}{\beta}\sum_{\omega_n}D(\omega_n) + q_{\rm EA}\right] - D^2(\omega_n)$$
$$-\frac{2y}{V^4}q_{\rm EA}^2D(-\omega_n) - \frac{2y}{V^4}\frac{q_{\rm EA}^2}{\beta}\sum_{\omega_1}D(\omega_1)D(-\omega_1-\omega_n)$$
$$-\frac{2y}{3V^4}\frac{1}{\beta^2}\sum_{\omega_1,\omega_2}D(\omega_1)D(\omega_2)D(-\omega_1-\omega_2-\omega_n) = 0, \qquad (11)$$

that closes the system of equations determining $q_{\rm EA}$ and $D(\omega_n)$. All dangerous terms proportional to β vanish because we have judiciously chosen q_{ab} equal to $q_{\rm EA}$ from the very beginning. Note that Eq. (11) does not contain W at all, while $q_{\rm EA} = 0$ only when $W = 0$, as can be seen from Eq. (10)

Electron glass phase. The saddle point solution of Eq. (8) in this phase must be characterized by the Parisi function $q(s)$ with $0 \leq s \leq 1$. Considerations, that are completely analogous to the classical case[5, 11, 15], lead to the functional form that has two plateaus:

$$q(s) = \begin{cases} q(0) = \left(\frac{3R_4}{4R_3}\right)^{\frac{1}{3}}, 0 < s < s_0 = \left(\frac{6R_4R_3^2}{R_2^3}\right)^{\frac{1}{3}} \\ \frac{R_2 s}{2R_3}, s_0 < s < s_1 = 1 - \sqrt{1 - \frac{4R_3R_1}{R_2^2}} \\ q_{\rm EA} = \frac{R_2 - (R_2^2 - 4R_1R_3)^{1/2}}{2R_3}, s_1 < s < 1. \end{cases} \qquad (12)$$

The function $q(s)$ in the above equation saturates at the value $q(1) = q_{\rm EA}$. This parametrization is in agreement with the definition $q_{\rm EA} = \max_{a\neq b}q_{ab}$. Combining Eqs. (9) and (12) we obtain that in the glassy phase

$$q_{\rm EA}^2 = -[D(0)V^4]/y, \qquad (13)$$

This equation connecting $D(0)$ and $q_{\rm EA}$ plays the same role as Eq. (10) in the Fermi liquid phase. We must substitute then the solution Eq. (12) into Eq. (8) and add subsequently the result to the remaining part of the action that contains $D(\omega_n)$. We will not write down the expression for the free energy density obtained after lengthy, but straightforward calculations, that are largely identical to those done in the Appendix C of Ref. [12]. We will state only that as a result of condition Eq. (13), there will be no terms in the free energy in which $D(0)$ and $q_{\rm EA}$ are coupled explicitly. Taking the variational derivative with respect to $D(\omega_n)$ results in the same Eq. (11), albeit in the glassy phase $q_{\rm EA}$ is connected with $D(0)$ by means of Eq. (13) rather than Eq. (10).

Approaching the glass transition. The necessity to obtain $D(0)$ from Eq. (11) leaves us with the task of its self-consistent solution. The exact analytical solution of this non-linear integral equation is clearly out of question. However, close to the $T = 0$ transition point the *leading* order of the correct solution is possible to obtain. The approximations we use hinge also upon the smallness of W and, consequently, $q_{\rm EA}$.

We notice first that, if $y = 0$, the complete solution is well-known to be[5, 6, 7] $D(\omega_n) = -\sqrt{|\omega_n| + \Delta}$, with Δ turning to zero right at the critical point. Let's assume that for $y \neq 0$ the leading approximation of $D(\omega_n)$ contains the same square root singularity as for $y = 0$, and analyze the role of the last two terms in Eq. (11). The key point in this analysis is the value of the integral

$$J(\Omega_m) = T\sum_{\omega_n} \sqrt{|\omega_n| + \Delta}\sqrt{|\omega_n + \Omega_m| + \Delta}, \qquad (14)$$

that at $T = 0$ is simply calculated to be:

$$J(\Omega) = \frac{1}{2\pi}\left\{\Lambda_\omega^{3/2} - \frac{\Omega^2}{4}\ln\frac{2\Lambda_\omega}{|\Omega|} + \frac{\Omega^2}{2}{\rm Arch}\frac{|\Omega| + 2\Delta}{|\Omega|} + \left(\frac{|\Omega|}{2} + \Delta\right)^2 \arcsin\frac{|\Omega|}{|\Omega| + 2\Delta} - (\Delta)^{3/2}\sqrt{|\Omega| + \Delta}\right\} \qquad (15)$$

In Eq. (15) Λ_ω denotes the upper critical cutoff of the order of unity. We see that all the terms in $J(\Omega)$, except the first one proportional to $\Lambda_\omega^{3/2}$, are of the order of $O(\Delta^2, \Omega^2)$. This means that the prelast term in Eq. (11) gives contributions that depend quadratically on small parameters Δ and ω_n, and, thus, subdominant to the leading terms that scale linearly with them. The cutoff-dependent part in its turn leads to a mere renormalization of the coefficient u in $uq_{\rm EA}$. We denote this renormalized term as $\tilde{u}q_{\rm EA}$. Inserting then $J(\omega_1 + \omega_n)$ to the last term of Eq. (11) and integrating over ω_2, we conclude, that the only effect this term produces is to renormalize the critical value r_c. This allows us to omit formally the last two integrals in Eq. (11) and resolve the ensuing quadratic equation for $D(\omega_n)$, obtaining that in the Fermi liquid phase:

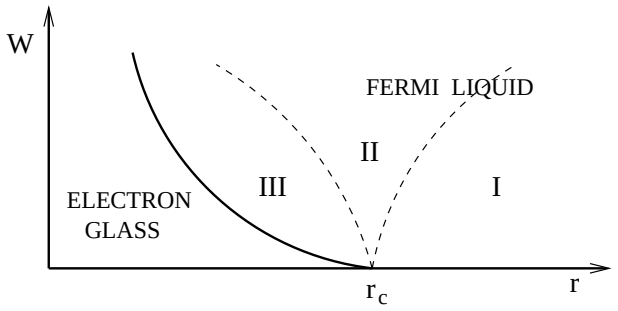

FIG. 1: Zero-temperature phase diagram demarcating the regions of Fermi liquid and electron glass phases. For small W, $r - r_c$ the curve separating these two phases scales roughly as $W \sim (r_c - r)^{3/2}$. The limiting behavior of the EA order parameter $q_{\rm EA}$ and gap Δ in different regimes is presented in the text.

$$D(\omega_n) = -\frac{yq_{\rm EA}^2}{V^4} - \sqrt{|\omega_n| + \Delta}, \qquad (16)$$

$$\Delta = r - r_c + \tilde{u}q_{\rm EA} \qquad (17)$$

in the leading approximation. It is easily verifiable *á posteriori* that the first term in Eq. (16), being inserted into the last two integrals of Eq. (11), renders the contributions of the higher order of smallness compared to $\tilde{u}q_{\rm EA}$. Together Eqs. (10) and (16) determine completely the $T = 0$ behavior of the disordered phase near the quantum critical point. As as a result of their solution, one can distinguish the following regimes on a $(r - r_c,\ W)$ plane, schematically depicted on Fig. (1).

(I) In this regime, in which $W \ll (r - r_c)^{3/4}$ and can be treated as a perturbation, $q_{\rm EA} = (W^2V^2)/4\sqrt{r - r_c}$, $\Delta \approx r - r_c$.

(II) This region is characterized by $|r - r_c|^{3/4} \ll W$. As a result, we have $q_{\rm EA} \approx (W^2V^2/4\sqrt{\tilde{u}})^{2/3}$ and $\Delta \approx \tilde{u}q_{\rm EA}$ in this regime.

(III) This regime, in which $(r_c - r)^{3/4} \gg W$, is the closest to the $T = 0$ critical boundary. EA order parameter, that crosses over to its value in the glassy phase, is given by $q_{\rm EA} = [(r_c - r)/\tilde{u}] + (\Delta/\tilde{u})$, with

$$\Delta = \left(\frac{2y(r_c - r)^2}{3\tilde{u}^2V^4} - \frac{W^2V^2\tilde{u}}{4(r_c - r)}\right)^2. \qquad (18)$$

From Eq. (18) it is easily seen that Δ vanishes at the critical line given by $W = (8y/3)\left[(r_c - r)/\tilde{u}V^2\right]^{3/2}$. This is exactly the line that determines the transition to the glassy phase obtained from the joint solution of Eqs. (10), (11) and (13). Taking into account Eq. (13), we resolve similarly the equation for $D(\omega_n)$ to get that below this line

$$D(\omega_n) = -\frac{yq_{\rm EA}^2}{V^4} - \sqrt{|\omega_n|}, \quad q_{\rm EA} = \frac{r_c - r}{\tilde{u}}. \qquad (19)$$

Within the spin glass phase, that obtains for all values of W, Δ is zero everywhere. Therefore, the imaginary part of the local dynamic susceptibility has the non-Fermi liquid singularity of the form $\sim \mathrm{sgn}(\omega)\omega^{1/2}$. The transition to the electron glass phase occurs as a second-order transition and is of the same character for both zero and non-zero on-site randomness W.

Finite temperature behavior. The evaluation of the temperature-dependent correction to the integral $J(\Omega_m)$ in Eq. (14) leads to the following results. If $\Delta \ll T$, the correction scales $\propto T^2$ for $|\Omega_m| \sim T$ and $\propto T^{3/2}|\Omega_m|$ for $|\Omega_m| \gg T$. If $\Delta \gg T$, it behaves $\propto T^2$ for $|\Omega_m| \sim T$ and $\propto T^2|\Omega_m|/\sqrt{\Delta}$ for $|\Omega_m| \gg T$. As a result, the equations governing the dependence of parameters $q_{\rm EA}(T)$ and $\Delta(T)$ at finite temperature can be obtained from those considered above by the formal substitution of r_c by $r_c(T) = r_c - c_1 T^2/\sqrt{\Delta}$ for $\Delta \gg T$, and $r_c(T) = r_c - c_2 T^{3/2}$ in the opposite limit. c_1 and c_2 are now, however, some complicated cutoff-dependent coefficients of the order of unity. Those modifications lead to the myriad of limiting cases close the quantum critical point that will not be enumerated here. Instead, we mention that only if T is small enough in the Fermi liquid phase, all temperature corrections behave as T^2 with some large prefactors arising because $W, |r - r_c| \ll 1$. On the contrary, for all temperatures in the glassy phase $q_{\rm EA}(T) = [r_c - r - c_2 T^{3/2}]/\tilde{u}$. The calculation of the electron self-energy[7] in the lowest order using Eqs. (5), (7) and (19) suggests, that the leading temperature correction to the elastic scattering rate is proportional to $T^{3/2}$ in the glassy phase. This gives rise to the non-Fermi liquid temperature dependence of the conductivity, $\delta\sigma \sim T^{3/2}$, in qualitative agreement with the experimental observations [4].

We emphasize that our results are based on the non-perturbative treatment of the RSB term in Eq. (6) and the specific conditions, connecting $q_{\rm EA}$ and q_{ab} [12]. An alternative saddle point solution of Eq. (6), in which $q_{\rm EA}$ and q_{ab} are allowed to be varied independently [5], leads the gapless glassy phase, only if $W = 0$, which we believe is incorrect. Our solution which has zero gap for *all* finite W, seems to be the one in agreement with the recent numerical simulations [16]. Though our theory is formulated for small W, the saddle-point Eqs. (11) and (13) are free from this parameter. This strongly suggests that the similar set of equations, with renormalized coefficients, should describe the glassy phase close to the transition for all W, including the limit $W \to \infty$ [3].

To conclude, we have presented a Landau theory description of the disordered Fermi liquid to electron glass quantum critical behavior. Our results represent an exact solution [18] of the model within a dynamical mean-field formulation, which is formally exact in the limit of large coordination. It is important to note that a glass transition having a character of a De Almeida-Thouless line [11], such as the one we describe, generally emerges within mean-field models. An alternative formulation [19], based on droplet approaches predicts the absence of such transitions for models with short-range interactions. In the case of an electron glass, the existence of the long-range Coulomb interaction opens a possibility that droplet approaches are not relevant, and that the glassy behavior of electrons could be well described using mean-field models. This possibility seems to find support in very recent experiments [20], which provide striking evidence of scale-invariant dynamical correlations inside the glass phase, consistent with the hierarchical picture of glassy dynamics as emerging from mean-field models.

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